Instanton and QCD-monopole Trajectory in the Abelian Dominating System

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ABSTRACT

Correlation between instantons and QCD-monopoles is studied in the abelian-gauge-fixed QCD. From a simple topological consideration, instantons are expected to appear only around the QCD-monopole trajectory in the abelian-dominating system. The QCD-monopole in the multi-instanton solution is studied in the Polyakov-like gauge, where $A_4(x)$ is diagonalized. The world line of the QCD-monopole is found to be penetrate the center of each instanton. For the single-instanton solution, the QCD-monopole trajectory becomes a simple straight line. On the other hand, in the multi-instanton system, the QCD-monopole trajectory often has complicated topology including a loop or a folded structure, and is unstable against a small fluctuation of the location and the size of instantons. We also study the thermal instanton system in the Polyakov-like gauge. At the high-temperature limit, the monopole trajectory becomes straight lines in the temporal direction. The topology of the QCD-monopole trajectory is drastically changed at a high temperature.

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1. Abelian Gauge Fixing and QCD-Monopole

Color confinement is one of the central issues in the nonperturbative QCD [1,2], and is characterized by the formation of the color-electric flux tube [2] with the string tension about 1GeV/fm. To understand the confinement mechanism, much attention has been paid for the analogy between the superconductor and the QCD vacuum [3-5] using the duality of the gauge theory. Similar to the superconductivity, the color-electric flux seems to be excluded in the QCD vacuum, which leads the formation of the squeezed color-flux tube between color sources. In this analogy, color confinement is brought by the dual Meissner effect originated from color-magnetic monopole condensation, which corresponds to Cooper-pair condensation in the superconductivity. As for the appearance of color-magnetic monopoles in QCD, 't Hooft [6] proposed an interesting idea of the abelian gauge fixing, which is defined by the diagonalization of a gauge-dependent variable $X(x)$. In this gauge, the nonabelian gauge theory like QCD is reduced into an abelian gauge theory with QCD-monopoles, which appear from the hedgehog-like configuration [7-11] corresponding to the nontrivial homotopy class on the nonabelian manifold, $\pi_2(\mathrm{SU}(N_c)/\mathrm{U}(1)^{N_c-1}) = Z_\infty^{N_c-1}$.

To begin with, the abelian gauge fixing is studied with attention to the ordering condition [10,11], which is closely related to the magnetic charge of QCD-monopoles. In general, the abelian gauge fixing consists of two sequential procedures.

1. The diagonalization of a gauge-dependent variable $X(x)$ by a suitable gauge transformation : $X(x) \rightarrow X_d(x)$ [7]. Since there also remains a discrete symmetry corresponding to the permutation of the diagonal elements of $X_d(x)$, the gauge group $\mathrm{SU}(N_c)_{\mathrm{local}}$ is reduced to $\mathrm{U}(1)^{N_c-1}_{\mathrm{local}} \times P^{N_c}_{\mathrm{global}}$ by the diagonalization of $X(x)$.

P^{N_c}-symmetry here becomes a global symmetry in the description with the continuous field theory, where local discrete changes are forbidden. On the other hand, such a P^{N_c}-symmetry often appears as a local symmetry in the lattice gauge theory

in case of the abelian gauge fixing without the ordering condition, which may lead to a problematic ultra-violet behavior of the field variable.

2. The ordering on the diagonal elements of $X_d(x)$ by imposing the additional condition, for instance,

$$X_d^1(x) \geq X_d^2(x) \geq ... \geq X_d^{N_c}(x). \tag{1.1}$$

The residual gauge group $\mathrm{U}(1)^{N_c-1}_{\mathrm{local}} \times P^{N_c}_{\mathrm{global}}$ is reduced to $\mathrm{U}(1)^{N_c-1}_{\mathrm{local}}$ by the ordering condition on $X_d(x)$ [10,11].

The magnetic charge of the QCD-monopole is closely related to the ordering condition in the diagonalization in the abelian gauge fixing [10,11]. For instance, in the SU(2) case, the hedgehog configuration as $X(x) = (\mathbf{x}\cdot\tau)$ and the anti-hedgehog one as $X(x) = -(\mathbf{x}\cdot\tau)$ provide a QCD-monopole with an opposite magnetic charge, "anti-QCD-monopole", because they are connected by the additional gauge transformation by

$$\Omega = \exp\{i\pi(\frac{\tau^1}{2}\cos\alpha + \frac{\tau^2}{2}\sin\alpha)\} \in P^2_{\mathrm{global}} \tag{1.2}$$

with an arbitrary constant α. Here, Ω physically means the rotation of angle π in the internal SU(2) space, and it interchanges the diagonal elements of $X_d(x)$, which leads a minus sign in the $\mathrm{U}(1)_3$ gauge field, $A_\mu^3(x)$. Thus, the magnetic charge of the QCD-monopole is settled by imposing the ordering condition on $X_d(x)$.

$P^{N_c}_{\mathrm{global}}$-symmetry is also important for the argument of gauge dependence. If a variable holds the residual gauge symmetry in the abelian gauge, it is proved to be $\mathrm{SU}(N_c)$ gauge invariant [12]. However, one should carefully examine the residual gauge symmetry, which often includes not only $\mathrm{U}(1)^{N_c-1}_{\mathrm{local}}$ but also $P^{N_c}_{\mathrm{global}}$. For instance, the dual Ginzburg-Landau theory [7] is, strictly speaking, an effective theory holding $\mathrm{U}(1)^{N_c-1}_{\mathrm{local}} \times P^{N_c}_{\mathrm{global}}$ symmetry. Hence, gauge dependence of a physical variable should be carefully checked in terms of the residual gauge symmetry $\mathrm{U}(1)^{N_c-1}_{\mathrm{local}} \times P^{N_c}_{\mathrm{global}}$ instead of $\mathrm{U}(1)^{N_c-1}_{\mathrm{local}}$ [12]. As a result, the dual gauge field $\vec{B}_\mu$ is

not SU(N_c)-invariant, because $\vec{B}_\mu$ is U$(1)^{N_c-1}$-invariant but is changed under the global P^{N_c} transformation.

We briefly compare the dual Higgs mechanism in the nonperturbative QCD vacuum with the ordinary Higgs mechanism. Like the Cooper pair in the superconductivity or the Higgs field in the standard theory [1], the charged-matter field to be condensed is the essential degrees of freedom for the Higgs mechanism. On the other hand, there is only the gauge field in the pure gauge QCD, and hence it seems difficult to find any similarity with the Higgs mechanism. In the abelian gauge, however, only the diagonal gluon behaves as the gauge field, and the off-diagonal gluon behaves as the charged-matter field, which leads QCD-monopoles as the relevant degrees of freedom for color confinement. Condensation of QCD-monopoles leads to mass generation of the dual gauge field through the dual Higgs mechanism [7,13], and therefore the QCD vacuum would be regarded as the dual superconductor by the abelian gauge fixing. In this framework, the nonperturbative QCD is mainly described by the abelian gauge theory with QCD-monopoles, which is called as the abelian dominance [6,14].

Many recent studies [11,12, 15-23] based on the lattice gauge theory have supported the realization of QCD-monopole condensation and the abelian dominance on the color confinement and other nonperturbative quantities of QCD in the maximally abelian gauge and/or in the Polyakov gauge. The crucial role of QCD-monopole condensation to the chiral-symmetry breaking is also supported by recent lattice studies [17,21,22] and the model analyses

[7-9,24,25].

In this paper, we study the relation between instantons [26] and QCD-monopoles in the abelian gauge. The instanton is also an important topological object [26] in the nonperturbative QCD appearing in the Euclidean 4-space corresponding to the nontrivial homotopy class on the nonabelian manifold, $\pi_3(\mathrm{SU}(N_c)) = Z_\infty$. If the system is completely described only by the abelian field, the instanton would lose the topological basis for its existence, and therefore it seems unable to survive

in the abelian manifold. However, even in the abelian gauge, nonabelian components remain relatively large around the QCD-monopoles, which are nothing but the topological defects, so that instantons are expected to survive only around the world lines of the QCD-monopole in the abelian-dominating system. The close relation between instantons and QCD-monopoles are thus suggested from the topological consideration.

2. QCD-monopoles in the Multi-Instanton System

We demonstrate a close relation between instantons and QCD-monopoles in the Euclidean SU(2) gauge theory in continuum [7-24]. Since there is an ambiguity on the gauge-dependent variable $X(x)$ to be diagonalized in the abelian gauge fixing [6,7], it would be a wise way to choose a suitable $X(x)$ so that the instanton configuration can be simply described. Here, we adopt the Polyakov-like gauge [10,11], where $A_4(x)$ is diagonalized. The Polyakov-like gauge has a large similarity to the Polyakov gauge, because the Polyakov loop $P(x)$ is also diagonal in this gauge.

Using the 't Hooft symbol $\bar{\eta}^{a\mu\nu}$, the multi-instanton solution is written as [26]

$$\begin{aligned} A^{\mu}(x) &= i\bar{\eta}^{a\mu\nu}\frac{\tau^a}{2}\partial^{\nu}\ln\phi(x) = -i\frac{\bar{\eta}^{a\mu\nu}\tau^a}{\phi(x)}\sum_k \frac{a_k^2(x-x_k)^{\nu}}{|x-x_k|^4}, \\ \phi(x) &\equiv 1+\sum_k \frac{a_k^2}{|x-x_k|^2}, \end{aligned} \tag{2.1}$$

where $x_k^{\mu} \equiv (\mathbf{x}_k, t_k)$ and a_k denote the center coordinate and the size of k-th instanton, respectively. In this case, one finds

$$A_4(x) = i\frac{\tau^a}{\phi(x)}\sum_k \frac{a_k^2(\mathbf{x}-\mathbf{x}_k)^a}{|x-x_k|^4}, \tag{2.2}$$

and therefore, near the center of k-th instanton, $A_4(x)$ takes a hedgehog configu-

ration around $\mathbf{x}_k$,

$$A_4(x) \simeq i\frac{\tau^a(\mathbf{x}-\mathbf{x}_k)^a}{|x-x_k|^2} \tag{2.3}$$

like a single-instanton solution. In the Polyakov-like gauge, $A_4(x)$ is diagonalized by a singular gauge transformation, which provides a QCD-monopole on the center of the hedgehog, $\mathbf{x} = \mathbf{x}_k$. Thus, the center of each instanton is inevitably penetrated by a QCD-monopole trajectory along the temporal direction in the Polyakov-like gauge [9,10,11]. In other words, instantons exist only along the QCD-monopole trajectories.

For the single-instanton system, $A_4(x)$ takes a hedgehog configuration around $\mathbf{x}_1$,

$$A_4(x) = ia_1^2\frac{\tau^a(\mathbf{x}-\mathbf{x}_1)^a}{(x-x_1)^2\cdot\{(x-x_1)^2+a_1^2\}}. \tag{2.4}$$

The diagonalization of $A_4(x)$ is carried out using a time-independent singular gauge transformation with the gauge function

$$\Omega(\mathbf{x}) = e^{i\tau_3\phi}\cos\frac{\theta}{2} + i(\tau_1\cos\alpha+\tau_2\sin\alpha)\sin\frac{\theta}{2} = \begin{pmatrix} e^{i\phi}\cos\frac{\theta}{2} & ie^{i\alpha}\sin\frac{\theta}{2} \\ ie^{-i\alpha}\sin\frac{\theta}{2} & e^{-i\phi}\cos\frac{\theta}{2} \end{pmatrix} \tag{2.5}$$

with θ and ϕ being the polar and azimuthal angles,

$$\mathbf{x}-\mathbf{x}_1 = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta). \tag{2.6}$$

Here, α is an arbitrary constant angle corresponding to the residual $\mathrm{U}(1)_3$ symmetry. Since $\Omega(\mathbf{x})$ is time-independent, $A_4(x)$ is simply transformed as

$$A_4(x) \to \Omega(\mathbf{x})A_4(x)\Omega^{-1}(\mathbf{x}). \tag{2.7}$$

After the singular gauge transformation by $\Omega(\mathbf{x})$, the abelian gauge field $A_\mu^3(x)$ has a singular part stemming from

$$A_\mu^{sing}(x) = \frac{1}{e}\Omega(\mathbf{x})\partial_\mu\Omega^{-1}(\mathbf{x}), \tag{2.8}$$

which leads to the QCD-monopole with the magnetic charge $g = 4\pi/e$ [7]. The

QCD-monopole appears at the center of the hedgehog, $\mathbf{x} = \mathbf{x}_1$, which satisfies $A_4(x) = 0$ in Eq.(2.4). Hence, the QCD-monopole trajectory $x^\mu \equiv (\mathbf{x}, t)$ becomes a simple straight line penetrating the center of the instanton as shown in Fig.1 (a),

$$\mathbf{x} = \mathbf{x}_1 \quad (-\infty < t < \infty), \tag{2.9}$$

at the classical level in the Polyakov-like gauge [7,9,10,11]. Similar relation for the QCD-monopole in a single instanton is found also in the maximally abelian gauge [27].

It should be noted that the singularity of $A_\mu(x)$ at the center of the instanton can be removed easily by a gauge transformation to the non-singular gauge [26], where the singular-free gauge field,

$$A_\mu(x) = i\frac{\tau^a(\mathbf{x}-\mathbf{x}_1)^a}{(x^2-x_1^2)+a_1^2}, \tag{2.10}$$

provides the same QCD-monopole trajectory as mentioned above. It is also worth mentioning that the QCD-monopole trajectory is not changed by the residual $\mathrm{U}(1)_3$-gauge transformation, so that QCD-monopoles in the Polyakov-like gauge are identical to those, *e.g.*, in the temporal gauge: $A_4(x) = 0$.

For the single anti-instanton system, one has only to replace $A_4(x) \to -A_4(x)$ corresponding to $\bar\eta^{a\mu\nu} \to \eta^{a\mu\nu}$ in the above argument [26]. Since this replacement interchanges the hedgehog and the anti-hedgehog on $A_4(x)$, it leads to the change of the QCD-monopole charge as mentioned in Section 2.1. Then, the anti-QCD-monopole with the opposite magnetic charge, $-g$, appears and passes through the center of the anti-instanton as shown in Fig.1 (b). In Figs.1 (a) and (b), relative difference on the QCD-monopole charge is expressed by the direction of the arrow.

For the two-instanton system, two instanton centers can be put on the zt-plane by a suitable spatial rotation in $\mathbf{R}^3$ without loss of generality, so that one can set $x_1 = y_1 = x_2 = y_2 = 0$. Owing to the axial-symmetry around the z-axis of the system, the QCD-monopole trajectory only appears on the zt-plane, and hence

one has only to examine $A_4(x)$ on the zt-plane by setting $x = y = 0$. In this case, $A_4(x)$ is already diagonalized on the zt-plane:

$$A_4(z,t;x=y=0) = i\frac{\tau^3}{\phi(z,t)}\sum_{k=1}^{2} a_k^2 \frac{(z-z_k)}{\{(z-z_k)^2+(t-t_k)^2\}^2} \equiv A_4^3(z,t)\tau^3. \quad (2.11)$$

Therefore, the QCD-monopole trajectory $x^\mu = (x, y, z, t)$ is simply given by $x = y = 0$ and $A_4^3(z,t) = 0$ or

$$\sum_{k=1}^{2} a_k^2 \frac{(z-z_k)}{\{(z-z_k)^2+(t-t_k)^2\}^2} = 0. \quad (2.12)$$

Here, $A_4(x)$ takes a hedgehog or an anti-hedgehog configuration near the QCD-monopole at each t [10,11]. The shape of the QCD-monopole trajectory depends only on the relative vector between the instanton centers and the ratio $a2/a1$ between the instanton sizes, which means the irrelevance of the absolute value of the instanton size. The typical scale of the system is mainly characterized by the relative distance between instantons, because the classical Yang-Mills theory has no scale-parameter.

We show in Figs. 2 (a),(b) and (c) the typical examples of the QCD-monopole trajectory in the two-instanton system. The QCD-monopole trajectories are found to be rather complicated even at the classical level. Fig.2 (a) shows the simplest case for two instantons with the same size, $a_1 = a_2$, locating at the same Euclidean time, $(z_1, t_1) = -(z_2, t_2) = (z_0, 0)$. In this case, the QCD-monopole trajectory (z, t) is analytically solved [9,10] as

$$z = 0 \quad \text{or} \quad t^2 = (z_0^2 - z^2) + 2|z_0|\sqrt{(z_0^2 - z^2)}, \quad (2.13)$$

and there appear two junctions and a loop in the QCD-monopole trajectory [9,10]. Here, the QCD-monopole charge calculated is expressed by the direction of the arrow. There also appears the anti-QCD-monopole at $z = 0$ for
$-\sqrt{3}|z_0| < t < \sqrt{3}|z_0|$.

Fig.2 (b) shows an example for two instantons with the same size, $a_1 = a_2$, but a little rotated in $\mathbf{R}^4$ as $(z_1, t_1) = -(z_2, t_2)$ =(1,0.05). In this case, the QCD-monopole trajectory has a folded structure [10,11]. Fig.2 (c) shows an example for two instantons locating at the same time $(z_1, t_1) = -(z_2, t_2)$=(1,0), but with a little different size, $a_2 = 1.1a_1$. There appears a QCD-monopole loop in this case [10,11]. Thus, the QCD-monopole trajectories originating from instantons are very unstable against a small fluctuation relating to the location or the size of instantons [10,11].

For a general N-instanton system with $N \geq 3$, it is rather difficult to find a suitable gauge transformation diagonalizing $A_4(x)$, and therefore it is hard to obtain the QCD-monopole trajectory. However, the QCD-monopole trajectory can be also obtained by $x = y = 0$ and $A_4(z, t) = 0$ as Eq.(2.12) for the multi-instantons located on the zt-plane: $x_k = y_k = 0$. We examine such a special case in the multi-instanton system, where $A_4(x)$ is given by

$$A_4(x) = \frac{i}{\phi(x)} \left((\tau_x x + \tau_y y) \sum_k \frac{a_k^2}{|x - x_k|^4} + \tau_z \sum_k \frac{a_k^2 (z - z_k)}{|x - x_k|^4} \right). \tag{2.14}$$

Near the QCD-monopoles $x_s^\mu \equiv (x_s, y_s, z_s, t_s)$ obeying $A_4(x_s^\mu)$ =0 with $x_s = y_s$=0, $A_4(x)$ becomes the hedgehog configuration at each time t_s,

$$\begin{aligned} A_4(x, y, z, t_s) &\simeq \frac{i}{\phi(x_s)} \left\{ (\tau_x x + \tau_y y) \sum_k \frac{a_k^2}{|x_s - x_k|^4} + \tau_z f(z; t_s) \right\} \\ &\simeq \frac{i}{\phi(x_s)} \left\{ (\tau_x x + \tau_y y) \sum_k \frac{a_k^2}{|x_s - x_k|^4} + \tau_z (z - z_s) \partial_z f(z_s; t_s) \right\}, \end{aligned} \tag{2.15}$$

where the function $f(z; t_s)$ is given by

$$f(z; t_s) \equiv \sum_k \frac{a_k^2 (z - z_k)}{\{(z - z_k)^2 + (t_s - t_k)^2\}^2}. \tag{2.16}$$

We examine the correspondence between QCD-monopoles and the function $f(z; t_s)$ at a fixed time t_s. The nodes of $f(z; t_s)$ provide the QCD-monopole trajec-

tory in the zt-plane: $f(z_s;t_s)=0$. Since the factor $\sum_k \frac{a_k^2}{|x_s-x_k|^4}$ is positive definite, the magnetic charge of the QCD-monopole depends only on the sign of the derivative $\partial_z f(z_s;t_s)$: For $\partial_z f(z_s;t_s)>0$, $A_4(x)$ becomes the hedgehog around x_s^μ, so that the QCD-monopole appears at x_s^μ. For $\partial_z f(z_s;t_s)<0$, $A_4(x)$ becomes the anti-hedgehog around x_s^μ, and therefore the anti-QCD-monopole with the opposite magnetic charge appears at x_s^μ. Thus, the nodes of $f(z;t_s)$ with the positive derivative provide the QCD-monopoles, and those with the negative derivative provide the anti-QCD-monopoles. We call hereafter the node with the positive derivative as the positive node, and that with the negative derivative as the negative node. Since the positive and negative nodes appear alternately in the continuous function $f(z;t_s)$, the QCD-monopole and the anti-QCD-monopole appear by turns spatially in the zt-plane. The function $f(z;t_s)$ has a definite asymptotic form as $z\to\pm\infty$,

$$f(z;t_s)\sim z^{-3}\sum_k a_k^2 \tag{2.17}$$

independent of t_s and the instanton centers, x_k^μ. Hence, the number of the positive node is one more than that of the negative node, which means that the total magnetic charge is always unity at each time t_s. The conservation law on the magnetic charge is thus guaranteed, which was also seen in Fig.2.

In general, the QCD-monopole trajectory becomes highly complicated and unstable in the multi-instanton system even at the classical level, and a small fluctuation of instantons often changes the topology of the QCD-monopole trajectory as shown in Fig.2. In addition, the quantum fluctuation would make it more complicated and more unstable, which leads to appearance of a long twining trajectory as a result. Hence, instantons may contribute to promote monopole condensation, which is signaled by a long complicated monopole loop in the lattice QCD simulation [12,18,19].

Very recently, the strong correlation between instantons and QCD-monopoles has been also supported in the lattice QCD simulations [10,11,21,28], and the

monopole dominance [11,12,15-23] for the topological charge has been pointed out both in the maximally abelian gauge [21] and in the Polyakov gauge [10,11] by investigating the instanton number after the decomposition of the abelian link variable into the singular (monopole-dominating) part and the regular (photon-dominating) part.

3. QCD-monopoles in the Instanton and Anti-instanton System

In this chapter, we study the QCD-monopole trajectory in the instanton and anti-instanton (I-$\bar{I}$) system within the dilute gas approximation. For the instanton and the anti-instanton located at x_1^μ and x_2^μ, respectively, one obtains

$$
\begin{aligned}
A^\mu(x) &= -i\frac{\bar{\eta}^{a\mu\nu}\tau^a}{\phi_{I\bar{I}}(x)}\left\{\frac{a_1^2(x-x_1)^\nu}{|x-x_1|^4} - \frac{a_2^2(x-x_2)^\nu}{|x-x_2|^4}\right\}, \\
\phi_{I\bar{I}}(x) &\equiv 1 + \frac{a_1^2}{|x-x_1|^2} - \frac{a_2^2}{|x-x_2|^2}
\end{aligned}
\tag{3.1}
$$

within the dilute gas approximation. Similarly to the previous chapter, one can put the centers of the instanton and the anti-instanton in zt-plane without loss of generality: x_1=y_1=x_2=y_2=0. The QCD-monopole trajectory only appears on the zt-plane, because of the axial-symmetry around the z-axis of the system, so that one has only to examine $A_4(x)$ on the zt-plane by setting $x = y = 0$. In this case, $A_4(x)$ is already diagonalized on the zt-plane:

$$
A_4(0,0,z,t) = \frac{-i\tau^3}{\phi_{I\bar{I}}(z,t)}\left\{\frac{a_1^2(z-z_1)}{\{(z-z_1)^2+(t-t_1)^2\}^2} - \frac{a_2^2(z-z_2)}{\{(z-z_2)^2+(t-t_2)^2\}^2}\right\}.
\tag{3.2}
$$

Hence, the QCD-monopole trajectory $x^\mu = (x, y, z, t)$ in the Polyakov-like gauge is simply given by $x = y = 0$ and $A_4(0, 0, z, t) = 0$ or

$$
\frac{a_1^2(z-z_1)}{\{(z-z_1)^2+(t-t_1)^2\}^2} - \frac{a_2^2(z-z_2)}{\{(z-z_2)^2+(t-t_2)^2\}^2} = 0.
\tag{3.3}
$$

Near the QCD-monopole, $A_4(x)$ takes a hedgehog or an anti-hedgehog configuration at each t. The centers of the instanton and the anti-instanton are penetrated by

the QCD-monopole and the anti-QCD-monopole, respectively. In Fig.3, we show the typical cases of the I-$\bar{I}$ system. The QCD-monopole trajectory is found to be simple two curves penetrating the centers of the instanton and the anti-instanton.

4. QCD-monopoles in the Thermal-Instanton System

We also study the thermal instanton system in the Polyakov-like gauge. The multi-instanton solution at finite temperature T is given by

$$A^{\mu}(x) = i\bar{\eta}^{a\mu\nu}\frac{\tau^a}{2}\partial^{\nu}\ln\phi_{_T}(x) = i\bar{\eta}^{a\mu\nu}\frac{\tau^a}{2}\partial^{\nu}\phi_{_T}(x)/\phi_{_T}(x), \tag{4.1}$$

where $\phi_{_T}(x)$ is a scalar function,

$$\begin{aligned}\phi_{_T}(x) &= 1+\sum_k a_k^2\sum_{n=-\infty}^{\infty}\frac{1}{(\mathbf{x}-\mathbf{x}_k)^2+(t-t_k-n/T)^2}\\ &= 1+\pi T\sum_k\frac{a_k^2}{|\mathbf{x}-\mathbf{x}_k|}\cdot\frac{\sinh(2\pi T|\mathbf{x}-\mathbf{x}_k|)}{\cosh(2\pi T|\mathbf{x}-\mathbf{x}_k|)-\cos\{2\pi T(t-t_k)\}}.\end{aligned} \tag{4.2}$$

In this system, $A_4(x)$ is given by

$$\begin{aligned}A_4(x) =& -i\frac{\pi T\tau^a}{2\phi_{_T}}\sum_k\frac{a_k^2(\mathbf{x}-\mathbf{x}_k)^a}{|\mathbf{x}-\mathbf{x}_k|^3}\Big(\frac{\sinh(2\pi T|\mathbf{x}-\mathbf{x}_k|)}{\cosh(2\pi T|\mathbf{x}-\mathbf{x}_k|)-\cos\{2\pi T(t-t_k)\}}\\ &-2\pi T|\mathbf{x}-\mathbf{x}_k|\cdot\frac{1-\cosh(2\pi T|\mathbf{x}-\mathbf{x}_k|)\cos\{2\pi T(t-t_k)\}}{[\cosh(2\pi T|\mathbf{x}-\mathbf{x}_k|)-\cos\{2\pi T(t-t_k)\}]^2}\Big)\end{aligned} \tag{4.3}$$

At the high-temperature limit $T\to\infty$,

$$A_4(x) \simeq -\frac{i\pi T}{2\phi_{_T}}\tau^a\sum_k\frac{a_k^2(\mathbf{x}-\mathbf{x}_k)^a}{|\mathbf{x}-\mathbf{x}_k|^3} \tag{4.4}$$

becomes time-independent, so that $A_4(\mathbf{x})$ can be diagonalized using a time-independent gauge transformation by $\hat{\Omega}(\mathbf{x})$,

$$A_4(\mathbf{x}) \to \hat{\Omega}(\mathbf{x})A_4(\mathbf{x})\hat{\Omega}^{-1}(\mathbf{x}) = A_4^d(\mathbf{x}), \tag{4.5}$$

where QCD-monopoles appear at the points $\mathbf{x}_s$ satisfying $A_4(\mathbf{x}_s)=0$, These points $\mathbf{x}_s$ includes all the centers of instantons, $\mathbf{x}_k$, and become the centers of the (anti-

)hedgehog configuration on $A_4(\mathbf{x})$. Thus, the QCD-monopole trajectory is reduced to simple straight lines

$$\mathbf{x} = \mathbf{x}_s \quad (-\infty < t < \infty), \tag{4.6}$$

where each instantons are penetrated in the temporal direction. Such a simplification of the QCD-monopole trajectory may corresponds to the deconfinement phase transition through the vanishing of QCD-monopole condensation [12,15-20,8,29].

For the thermal two-instanton system, all instanton centers can be put on the zt-plane by a suitable spatial rotation in $\mathbf{R}^3$ like the two-instanton system at $T = 0$, so that one can set as $x_k = y_k = 0$ $(k = 1, 2)$. Owing to the axial-symmetry around the z-axis of the system, the QCD-monopole trajectory only appears on the zt-plane, where $A_4(x)$ in Eq.(4.3) is already diagonalized. Hence, the QCD-monopole trajectory $x^\mu = (x, y, z, t)$ is simply given by $x = y = 0$ and $A_4(z, t; x = y = 0) = 0$. Here, $A_4(x)$ takes a hedgehog or an anti-hedgehog configuration near the QCD-monopole at each t.

We show in Fig.4 the typical examples of the QCD-monopole trajectory in the thermal two-instanton system. As temperature goes high, the trajectory tends to be straight lines in the temporal direction. There also appears the QCD-monopole with the opposite magnetic charge at the point satisfying $A_4(x) = 0$. The topology of the QCD-monopole trajectory is drastically changed at $T_c \simeq 0.6d^{-1}$, where d is the distance between the two instantons. Since the classical Yang-Mills theory has no scale-parameter, the typical scale of the system is mainly characterized by the relative distance d. If one adopts $d \sim 1$fm as a typical mean distance between instantons, such a topological change occurs at $T_c \sim 120$MeV [10,11].

5. Summary and Concluding Remarks

We have studied the physical meanings of the abelian gauge fixing in terms of the analogy between the nonperturbative QCD vacuum and the superconductor. In the abelian gauge, the QCD-monopole appears from the hedgehog configuration corresponding to the homotopy group $\pi_2(\mathrm{SU}(N_c)/\mathrm{U}(1)^{N_c-1}) = Z_\infty^{N_c-1}$. The ordering condition in the abelian gauge fixing is important for the determination of the magnetic charge of the QCD-monopoles.

We have studied the relation between instantons and monopoles in the abelian gauge. Simple topological consideration indicates that instantons survive only around the world line of the QCD-monopole, which is the topological defect.

We have found a close relation between instantons and the QCD-monopole trajectory in the Polyakov-like gauge, where $A_4(x)$ is to be diagonalized. Every instantons are penetrated by the world lines of QCD-monopoles. Anti-instantons are penetrated by the anti-QCD-monopoles with the opposite magnetic charge. The QCD-monopole trajectory in $\mathbf{R}^4$ tends to be folded and complicated in the multi-instanton system, although it becomes a simple straight line in the single-instanton solution. The QCD-monopole trajectory is very unstable against a small fluctuation on the location and the size of instantons. We have found that the magnetic charge conservation is guaranteed by the argument on the node property of a continuous function relating to $A_4(x)$.

We have studied the QCD-monopole trajectory in the instanton and the anti-instanton system in the Polyakov-like gauge. The QCD-monopole trajectory becomes simple two curves penetrating the instanton and the anti-instanton.

We have also studied the thermal instanton system in the Polyakov-like gauge. At the high-temperature limit, the QCD-monopole trajectory becomes simple straight lines in the temporal direction. The QCD-monopole trajectory drastically changes its topology at a high temperature.

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FIGURE CAPTIONS

1) The QCD-monopole trajectory (a) in the single-instanton system, (b) in the single anti-instanton system. The (anti-)instanton is denoted by a small circle.

2) Examples of the QCD-monopole trajectory in the two-instanton system with (a) $(z_1, t_1) = -(z_2, t_2)$ =(1,0), $a_1 = a_2$; (b) $(z_1, t_1) = -(z_2, t_2)$ = (1,0.05), $a_1 = a_2$; (c) $(z_1, t_1) = -(z_2, t_2) = (1, 0), a_2 = 1.1a_1$.

3) The QCD-monopole trajectory in the system of the instanton and anti-instanton located at $(0, 0, z_1, t_1)$ and $(0, 0, z_2, t_2)$, respectively. The parameters are taken as (a) $(z_1, t_1) = -(z_2, t_2)$ =(1,0), $a_1 = a_2$; (b) $(z_1, t_1) = -(z_2, t_2) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, $a_1 = a_2$; (c) $(z_1, t_1) = -(z_2, t_2) = (1, 0), a_2 = 2a_1$.

4) The QCD-monopole trajectory in the thermal two-instanton system with $(z_1, t_1) = -(z_2, t_2) = (d/2, 0)$ and $a_1 = a_2$ (a) at $T^{-1} = 2d$; (b) at $T^{-1} = 1.5d$. The same with $(z_1, t_1) = -(z_2, t_2) = (d/2, 0.05d/2)$ and $a_1 = a_2$ (c) at $T^{-1} = 2d$; (d) at $T^{-1} = 1.5d$. The drastic change of the QCD-monopole trajectory is found between the low-temperature ($T^{-1} = 2d$) and the high-temperature ($T^{-1} = 1.5d$) cases.